**ARCH and GARCH**

**Conditional Heteroscedastic Models of Volatility:**

The prime motivation behind the development of conditional volatility models is twofold. First, the linear time series models were inappropriate in the sense that they provide poor forecast intervals, and it was contended that like conditional mean, variance (volatility) could as well evolve over time, and hence it was important to model them both simultaneously. Secondly, an assumption of Classical Linear Regression Model (CLRM) is that the variance of the error term is constant. If the errors are heteroscedastic, but assumed to be homoscedastic, an important implication would be that standard error estimates could be wrong. It is unlikely in the context of financial time series that the variance are constant over time and it makes sense to consider a model that does not assume that variance is constant. An attempt in this regard was made by Engel (1982) who proposed the Auto Regressive Conditional Heteroscedastic (ARCH) model. Another important feature of many series of financial asset returns which provides a motivation for the ARCH class of models is known as “volatility clustering” or “volatility pooling”. This volatility clustering describes the tendency of large changes in asset prices (of either sign) to follow large changes, and small changes (of either sign) to follow small changes. Hence the current level of volatility tends to be positively correlated with its level during the immediate preceding periods.

**The ARCH Model**

The first model that provides a systematic framework for volatility modelling is the ARCH model of Engle (1982). The model shows that it is possible to simultaneously model the mean and variance of a series. As a preliminary step to understand Engle’s methodology, let’s estimate a stationary ARMA model 

where  for all t and forecast.

A forecast of is conditional expectation of in period t, given the value of  as follows  Since E[εt+1] = 0 (5.1)

If we use this conditional mean to forecast , the forecast error variance is 

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Instead, if unconditional forecasts are used, the unconditional forecast is always the long run mean of the sequence that is equal to . The unconditional forecast error variance is

 (5.2)

Since, the unconditional forecast has a greater variance than the conditional forecast. Thus, conditional forecasts are preferable. Similarly, if the variance of  is not constant, we can estimate any tendency for sustained movements in the variance using an ARMA model. Let’s will be the estimated residuals from the model, so that the conditional variance of will be

. (5.3)

Thus far, we have set equal to. Now suppose that the conditional variance is not constant. One simple strategy is to model the conditional variance as an AR (q) process using the square of the estimated residuals:

 (5.4)

Where  is a white noise process. If the values of all equal to zero, the estimated variance is simply the constant. Otherwise the conditional variance of evolves according to the autoregressive process given by (5.4). As such we can use (5.4) to forecast the conditional variance at t+1 as



For this reason, an equation like (5.4) is called an autoregressive conditional heteroscedastic (ARCH) model.

Though the ARCH model offers some advantages described above, it has some weakness too:

* The model assumes that positive and negative shocks have the same effect in volatility because it depends on the square of previous shocks. In practice it is well known that price of the financial asset responds differently to positive and negative shocks.
* The ARCH model is rather restrictive. For instance, of an ARCH (1) model must be in the interval [0, 1/3] if the series is to have a finite fourth moment. The constraints become complicated for higher order ARCH model.
* The ARCH model does not provide insights for understanding the source of variations of a financial time series. It only provides a mechanical way to describe the behaviour of the conditional variance. It gives no indication about what causes such behaviour to occur.
* ARCH models are likely to over predict the volatility because they respond slowly to large isolated shocks to the return series.

**Building an ARCH Model:**

A way to build an ARCH model consists of three steps. Step (1) builds an econometric model for example an ARMA model for the return series to remove any linear dependence in the data and use the residual series of the model to test for ARCH effects. Step (2) specifies the ARCH order and performs estimation. Step (3) involves checking the fitted ARCH model carefully and refining it if necessary.

**The GARCH Model**

GARCH models explain variance by two distributed lags, one on past squared residuals to capture high frequency effects or news about volatility from the previous period measured as the lag of the squared residual from mean equation, and second on lagged values of variance itself to capture long term influences. In the GARCH (1, 1) model, the variance expected at any given data is a combination of long run variance and the variance expected for the last period, adjusted to take into account the size of the last periods observed shock. In the GARCH model estimates for financial asset returns data, the sum of coefficients on the lagged squared error and lagged conditional variance is very close to unity. This implies that shocks to the conditional variance will be highly persistence and the presence of quite long memory but being less than unit, it is still mean reverting.

Bollerslev (1986) proposes a useful extension of ARCH model known as the Generalized ARCH (i.e. GARCH) model. Bollerslev extended Engle’s original work by developing a technique that allows the conditional variance to be an ARMA process. Let the error process be such that

where and 

 (5.5)

Since is a white noise process that is independent of past realization of , the conditional and unconditional means of are equal to zero. By taking the expected values of, it is easy to verify that. The important point is that the conditional variance of is given by. Thus, the conditional variance of is given by in equation (5.5).

The generalised ARCH (p, q) model of equation (5.5) is called as GARCH (p, q) that allows for both autoregressive and moving average components in the heteroskedastic variance. If we set *p = 0* and *q = 1*, it is clear that the first order ARCH model is simply a GARCH (0, 1) model. If all the equal to zero, the GARCH (p, q) model is equivalent to an ARCH (q) model. The benefit of GARCH model should be clear as a higher order ARCH model may have a more parsimonious GARCH representation which is much easier to identify and estimate. This is particularly true since all coefficients in (5.5) must be positive.

ARCH component α (alpha) reflects the influence of random deviations in previous period error terms on σ which is a function of random error terms and realized variance of previous periods. Similarly, GARCH coefficient β (beta) measures the part of the realized variances in the previous period that is carried over in to the current period. The sum of ARCH coefficient and GARCH coefficient (α + β) determines the short run dynamics of the resulting volatility time series. More specifically, a large ARCH error coefficient (α) means that volatility reacts intensely to market movements and a large GARCH error coefficient (β) indicates that shocks to conditional variance take a long time to die out. So volatility is persistence. Hence current volatility can be explained by past volatility that tends to persist overtime. If α is relatively high and β is relatively low, then volatility tends to be spikier.